# Feedback Mechanism of Low-Speed Edgetones

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A feedback mechanism of low-speed edgetones is analyzed by using the jet-edge interaction model in which reaction of the edge is regarded as an array of dipoles. From the jet-edge interaction model the surface pressure of the edge and the upstream wave are estimated by assuming the downstream disturbance as a sinuously oscillating flow with a constant convection speed. The surface pressure distribution on the edge is found to increase from zero at the tip to a peak value around a quarter wavelength downstream, which may be regarded as an effective source point of the upstream-propagating sound wave. From the condition that the two wave trains should be phase-locked at the nozzle lip, the phase factor $p$ is found to be in the range, $-1/2<p<0$, in the phase criterion of the form, $\frac{h}{\Lambda}+\frac{h}{\lambda}=n+p$, where h is the stand-off distance, $\Lambda$ and $\lambda$ the wavelengths of downstream and upstream respectively, and $n$ the stage number. From the existing experimental data the phase factor is estimated on the basis that the convection wavelength is dependent only upon the mean jet velocity and the frequency, but not upon the stage number. The experimental values are in the same range of the theoretical estimation indicating that the phase factor is not a universal constant but varies in the range, $-1/2<p<0$, depending on the nozzle-edge configuration and the flow condition.

**Key Words:** Edgetone, Feedback, Phase, Jet-Edge, Impinging Jet

## 1. Introduction

Self-sustained shear-layer oscillations, responsible for whistling flow noise and undesirable structural loading, have been observed for a wide variety of shear-layer impingement configurations as reviewed by Rockwell(1983). The edgetone, a typical phenomenon of these oscillations, is produced by an edge placed in the path of a plane jet issuing from a two-dimensional nozzle of high aspect ratio.

When the free shear layer near the nozzle lip is excited, a disturbance is initiated and convected downstream and amplified, if unstable, into organized vortices. When the vortices impinge on the edge, pressure waves are generated and propagate upstream to the nozzle lip to produce another disturbance as shown in Fig. 1. The upstream-propagating sound and the downstream-convected flow constitute a feedback loop and should be phase-locked at the nozzle lip in order to close the loop. Such a feedback theory was proposed first by Powell(1953) in the following form,

$$\frac{h}{\Lambda}+\frac{h}{\lambda}=n+p \tag{1}$$

where $h$ is the stand-off distance between the nozzle lip and the edge tip, $\Lambda$ and $\lambda$ are the wavelengths of the downstream-propagating and the upstream-propagating waves respectively, $n$ is an integer representing sawtooth-like stages of frequencies, and $p$ is a nonintegral number for the phase factor associated with the possible phase delay in the process of jet-edge interaction.

Powell(1953) proposed $p=1/4$. Although it has never been justified rigorously, $p=1/4$ has been regarded as reasonable because it is in agreement with the experimental data obtained by Brown(1935) through smoke-visualization exper-

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iment. Since then, investigations were focussed on the develpment of theoretical model for the phase criterion.

Holger *et al.* (1977) analyzed the jet-edge interaction mechanism by considering a flat plate placed between a fully developed infinite vortex street. The phase factor $p$ was estimated by calculating the potential flow induced at the nozzle lip due to the convecting vortices and by imposing the phase-locking condition. Although they obtained $p=-1/4$ in addition to some positive values different depending on stages, they discarded $p=-1/4$, as it is not in the range of Brown's data.

Crighton(1992) performed a linear analysis to predict the frequency characteristics of the feedback cycle. Inviscid flow equations with vortex-sheet shear layers and aligned flat-plate boundaries were solved asymptotically by Wiener-Hopf methods. The phase factor was found to be $p=-3/8$, and the difference from $p=1/4$ of Powell's model was attributed to the boundary condition associated with the edge and nozzle configurations.

Recently, however, the present author(Kwon[a,b] 1996) proposed $p=-1/4$ based on the analysis of existing experimental data. He showed that Brown's data obtained by visualization experiment for low-speed jets are not appropriate to be compared with the phase criterion model of the form of Eq. (1). Although the model is based on the condition that the convection speed is constant along the jet, the experiment was performed at such low speeds that the convection speed decreased with distance.

Therefore, the objective of this study is to elaborate the feedback mechanism of edgetones based on the proposed model of the athor(Kwon [b]1996), in which $p=-1/4$ was verified. Generation of the upstream wave due to the edge is analyzed by the simple model proposed by Kwon and Powell(1992, 1994) and Kwon[b](1996) for the jet-edge interaction, where the reaction of edge is regarded as an array of dipoles and the downstream disturbance as a sinuously fluctuating flow with a constant convection speed. The boundary effect of the plate is taken into account by attenation constant of the downstream disturbance amplitude. By imposing boundary conditions on the edge surface, the strength of dipole is to be obtained. Using the dipoles the pressure distribution on the edge surface and the upstream velocity are estimated. Then the phase criterion is obtained by the phase- locking condition at the nozzle lip and found to be in the range of $-1/2<p<0$, instead of a constant $p=-1/4$, depending upon the attenuation constant. Finally the factor is deduced from the existing experimental data and discussed in comparison with the theoretical model.

## 2. Jet-Edge Interaction

The present model for jet-edge interaction is that proposed by Kwon and Powell(1992) and Kwon[b](1996), where the edge reacting to the impinging jet has been replaced by an array of dipoles, in consideration of antisymmetry of the unstable jet about the central plane, as shown in Fig. 1. The edge with small wedge angle is regarded as a flat plate divided into small elements acting as dipole sources. Then, if the incident flow driven by impinging vortices is given, the strengths of the dipoles responding to this flow can be estimated by solving a set of simultaneous equations to be obtained by imposing boundary conditions on the surface of each element. The downstream-convected flow impinging on the edge can be regarded for simplicity as a sinuously fluctuating flow. Then the complex amplitude of the transverse velocity impinging on the edge surface $V_{im}$ can be expressed by

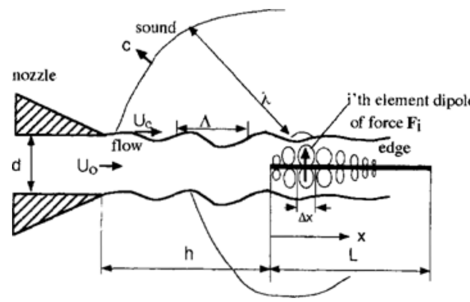


**Fig. 1** Edgetone system modelled by an array of dipoles.

$$V_{im}(x) = V_0 \exp[-(\alpha+2\pi i)x/\Lambda] \quad (2)$$

where $V_0$ is the amplitude at the tip. $\Lambda$ the wavelength of the sinuosity, $i$ the imaginary number $(-1)^{1/2}$ and $x$ the downstream distance from the edge tip. The constant $\alpha$ represents a parameter to take into account the amplitude attenuation due to the viscous damping or the generation of secondary vortex with boundary separation in the process of interaction.

The edge, simplified as a thin plate of length L, is divided into dipole elements with force of complex amplitude $F_i$, as shown in Fig. 1. The complex amplitude of transverse velocity induced, by the i' th dipole element placed at coordinate $x_i$, on the central plane of jet $(x<0)$ or the edge surface $(x>0)$, $V_{r,i}$, can be obtained by the Hankel function $H_1$ of the second kind as

$$V_{r,i}(x) = \frac{F_i}{2\rho c|x-x_i|} H_1(k|x-x_i|) \quad (3)$$

where $\rho$ is the density, $c$ the sound velocity and $k$ the acoustic wave number, $k=2\pi/\lambda$. Then the flow driven by the edge in response to the impinging flow, $V_r(x)$, should be the sum of $V_{r,i}(x)$ for all the elements. That is,

$$V_r(x) = \sum_{i=1}^{N} V_{r,i}(x) \quad (4)$$

Then the complex amplitude of resultant transverse velocity $V(x)$ can be obtained by superimposing the flow induced by the edge upon the incident flow as

$$V(x) = V_{im}(x) + V_r(x) \quad (5)$$

Now the force amplitude $F_i$ of each dipole element is to be obtained simultaneously from the boundary conditions imposed on each element. Here the integral form of rigid wall condition has been used as

$$\int_j V(x)\,dx = 0,\ j=1, 2, 3, \cdots, N \quad (6)$$

Substituting the equations from (2) to (5) into Eq. (6) yields the following integral equation.

$$\sum_{i=1}^{N} \frac{F_i}{2\rho\omega} \int_{x_i-\Delta x/2}^{x_i+\Delta x/2} \frac{kH_1(k|x-x_i|)}{|x-x_i|} dx = -V_0 \int_{x_i-\Delta x/2}^{x_i+\Delta x/2} \exp^{-(\alpha+2\pi i)x/\Lambda} dx,\ j = 1, 2, 3, \cdots, N \quad (7)$$

where the left-side term means the upward volume flow rate through the j'th element to compensate for the downward volume flow rate by the incident jet, the right-side term. The above equation can be simplified in terms of a linear equation by

$$a_{ij} F_i = b_j \quad (8)$$

where

$$a_{ij} = \frac{1}{2\rho\omega} \int_{x_i-\Delta x/2}^{x_i+\Delta x/2} \frac{kH_1(k|x-x_i|)}{|x-x_i|} dx \quad (9)$$

$$b_j = -V_0 \int_{x_i-\Delta x/2}^{x_i+\Delta x/2} \exp^{-(\alpha+2\pi i)x/\Lambda} dx \quad (10)$$

The coefficient $a_{ij}$ is associated with the contribution of the source at the $i$'th element to the volume flow rate on the $j$'th element. When $i=j$, the integral contains the singular source point to be expressed by

$$a_{ii} = \frac{1}{2\rho\omega} \int_{-\Delta x/2}^{\Delta x/2} \frac{kH_1(k|\xi|)}{|\xi|} d\xi \quad (11)$$

At $\xi=0$, there is the dipole source and the integrand of Eq. (11) relevant to the flow driven by this source is not defined. Hence the direct integration of the above equation is impossible. However, since $k\Delta x/2 \ll 1$, the integrand can be approximated by $\frac{kH_1(k|\xi|)}{|\xi|} \approx -\frac{2i}{\pi\xi^2}$, so that Eq. (11) is to be simplified by

$$a_{ii} = \frac{-i}{\rho\omega} \int_{-\Delta x/2}^{\Delta x/2} \frac{1}{\pi\xi^2} d\xi \quad (12)$$

The above equation shows that the flow field near the source is independent of the wave number, that is, the fluid can be treated as incompressible, as to be expected. For incompressible fluid, the volume flow rate, across an infinite plane, driven by the source should be equal to that of the source itself in the opposite direction, that is,

$$\int_{-\infty}^{\infty} \frac{1}{\pi\xi^2} d\xi = 0 \quad (13)$$

Thus $a_{ii}$ of Eq. (11) can be estimated by

$$a_{ii} = \frac{2i}{\rho\omega} \int_{\Delta x/2}^{\infty} \frac{1}{\pi\xi^2} d\xi = \frac{4i}{\rho\omega\pi\Delta x} \quad (14)$$

Now numerical solution can be performed for Eq. (8) with a suitable element size $\Delta x$. Once the dipole force is obtained, the complex amplitude of surface pressure $P_s$ on the upper side of the edge element is to be estimated by the relationship

$$P_s(x_i) = \frac{F_i}{2\Delta x} \tag{15}$$

where division by 2 is associated with the antisymmetry of pressure distribution between the upper and lower surfaces of the edge. The reliability of the numerical solution might be checked by the surface pressure. When the number of elements per wavelength of the downstream-propagating disturbance is more than 20, that is, $\Delta x < \Lambda/20$, the surface pressure distribution on the edge became independent of further reductions in $\Delta x$. The effect of the compressibility on the surface pressure on the edge near the tip was negligibly small implying that the surface reaction is determined just by the incompressible flow field.

Figure 2 shows the magnitude of the complex amplitude of surface pressure on the edge for various attenuation constants $\alpha$, for an edge length of $L=5\Lambda$. The surface pressure amplitude $P_s$ has been normalized by $\rho V_0 U_c$, where $U_c$ is the convection speed of the downstream disturbance, which is the pressure induced when an infinitely extended plate vibrates transversely with amplitude $V_0$ and a subsonic wave speed $U_c$. It is shown that the pressure is zero at the edge tip and maximum at a little distance downstream as has been observed by Kakayoglu and Rockwell (1986). This feature is to be explained by the antisymmetry of the downstream disturbance about the central surface of jet, which is the physical basis of the present dipole model. It is noted that the point for the maximum pressure shifts from $1/2\Lambda$ downstream to the tip as the attenuation constant $\alpha$ increases.

In addition, we can see in Fig. 2 that, although the surface pressure decreases with the attenuation constant $\alpha$, its maximum value is almost equal to $\frac{1}{2}$ in dimensionless form, that is,

$$P_{s,\max} \simeq \frac{1}{2}\rho V_0 U_c \tag{16}$$

which means that the surface pressure on the edge is almost same as half the pressure on a plate vibrating transversely with amplitude $V_0$ and subsonic phase speed $U_c$.

Figure 3 shows the point for the maximum surface pressure, $x_{\max}$, as a function of the edge length for different convection Mach numbers defined by $M_c = \frac{U_c}{c}$, with $\alpha=2$. When the edge is short enough, that is, $L \ll \frac{\Lambda}{2}$, the peak occurs at the center, $x_{\max} = \frac{L}{2}$, as to be expected by considering that the reaction of both ends of the edge is zero. In addition, it is to be noted that the surface pressure distribution is independent of the convection Mach number up to 0.1, suggesting that the effect of the compressibility of fluid on the edge reaction is negligible in the case of low-speed edgetones.

Since the sound radiation by the edge is proportional to the surface pressure, the maximum pressure point can be regarded as the effective source point, although the exact point can be different

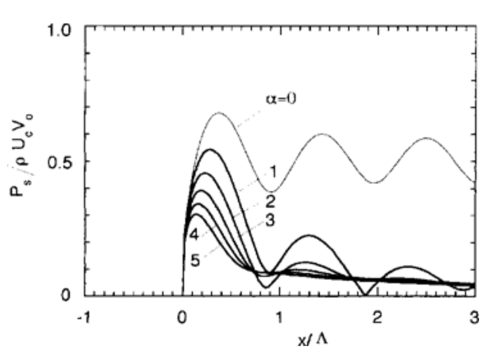


**Fig. 2** Surface pressure distribution on the edge.

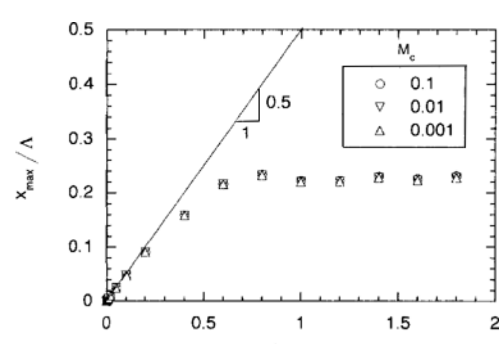


**Fig. 3** Peak pressure point on the edge surface as a function of the edge length, normalized by the convection wavelength, for $\alpha=2$.

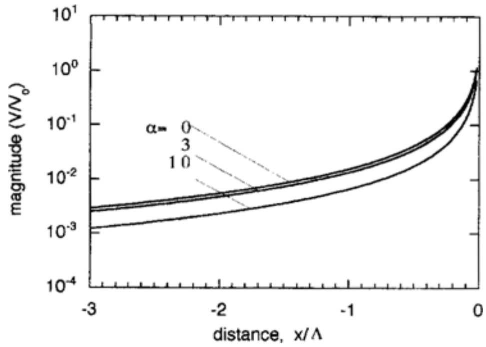


**Fig. 4** Amplitude variation of the transverse velocity of the upstream disturbance with the distance upstream from the edge tip.

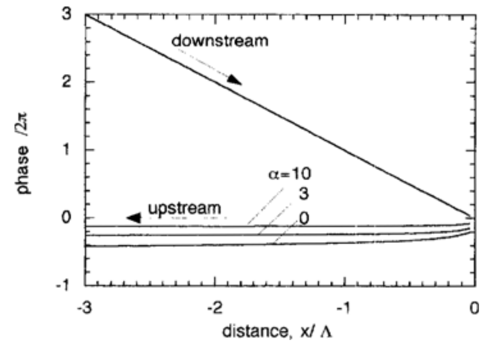


**Fig. 5** Phase variation of the upstream and downstream disturbances with the upstream distance.

somewhat depending on the radiating direction. The effective source point approaches nearer to the edge tip as the attenuation constant $\alpha$ increases if the edge is not so short compared with the convection wavelength. Such feature about the source of sound radiation has been observed by many investigators as Kaykayoglu and Rockwell (1986), but there has been no analysis associated with this point.

## 3. Upstream Disturbance and Phase Criterion

Transverse velocity of the upstream disturbance due to the jet-edge interaction may be estimated by superimposing the flow by each dipole element, based on Eq. (4). Figure 4 shows the magnitude of complex amplitude of the transverse velocity for the jet whose convection speed is very slow compared with the sound speed. It is shown that the amplitude differs somewhat depending on the attenuation constant $\alpha$ and decreases steeply in the vicinity of the edge tip, then slowly with distance along the upstream. Figure 5 shows the phase variation of the upstream disturbance $V_r(x)$, propagating at the sound speed, in comparison with that of the downstream disturbance $V_{im}(x)$, propagating at the convection speed. The downstream phase decreases linearly in proportion to the propagating distance but the upstream phase is almost constant because the jet speed was assumed to be low enough to regard the upstream sound speed c is infinite. In the vicinity of the edge tip, however, the upstream phase shows some variation due to the interference among the distributed dipole sources having different phases.

For both the upstream and the downstream disturbance to constitute a feedback loop between the nozzle lip and the effective source point of the edge, as described by Powell (1953), two conditions for phase and gain must be satisfied. First, the nozzle should be placed where the phase difference between the upstream and downstream disturbances is $2\pi n$ with positive integers n. Second, the amplitude of downstream disturbance must be amplified enough to compensate for the decay through the upstream propagation. The following sections are concerned primarily with the phase condition. Since the estimation of the upstream decay is possible by the present model, the onset condition of the edgetone can be found if only the amplification of downstream-convected vortices is known. But the amplification of unstable shear layer is not known yet and the second condition will not be dealt with here.

When the jet speed is low the ratio of the convection to the acoustic wavelength $\Lambda/\lambda$ is negligibly small, so that the phase-locking condition of Eq. (1) can be simplified as $h/\Lambda = n + p$. Then the phase factor $p$ can be obtained by $h/\Lambda$ corresponding to the phase difference $2n\pi$ between upstream and downstream in Fig. 5 and the result is shown in Fig. 6. It is found here that $p$ is not a constant but varies depending on the attenuation constant $\alpha$, while its dependence on the stage n is negligibly small, in the range

$$-\frac{1}{2}<p<0 \qquad (17)$$

The present author (Kwon[a] and Kwon[b] 1996) showed that $p=-1/4$ is reasonable by checking its physical properness that the wavelength should be dependent upon both frequency and jet velocity, but not upon the stage number $n$, based on the existing experimental data. However, since this result has been obtained from experimental data of limited range of jet velocities, examination of various experimental data is necessary.

In this paper, the experimental value of $p$ is obtained by estimating the wavelength as a function of $p$ according to Eq. (1), based on the experimental data about the relationship between $h$ and $n$ for the same frequency at a fixed jet velocity. Since the convection speed $U_c$ or wavelength $\Lambda$ should depend only upon the mean jet velocity $U_0$ and the frequency $f$, $\Lambda$ should be independent of stage $n$ for the same value of $U_0$ and $f$. The experimental data are those of Powell and Unfried (1964). The length of the parallel channel was 140mm and the edge was made of a sharp edged brass wedge having a wedge angle of 30°.

Figure 7 shows the estimated convection wavelength normalized by the nozzle width as a function of $p$ for the data measured using a slit nozzle of width 1.04mm and breadth 12.7mm. It is shown

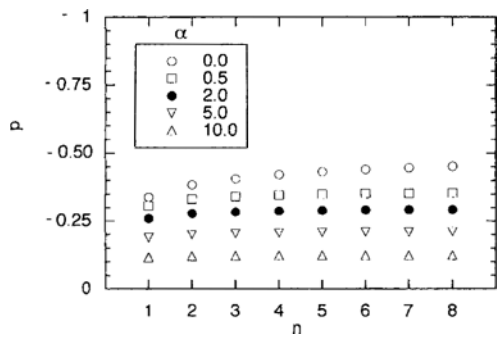


**Fig. 6** Phase factor p estimated for various attenuation constants.

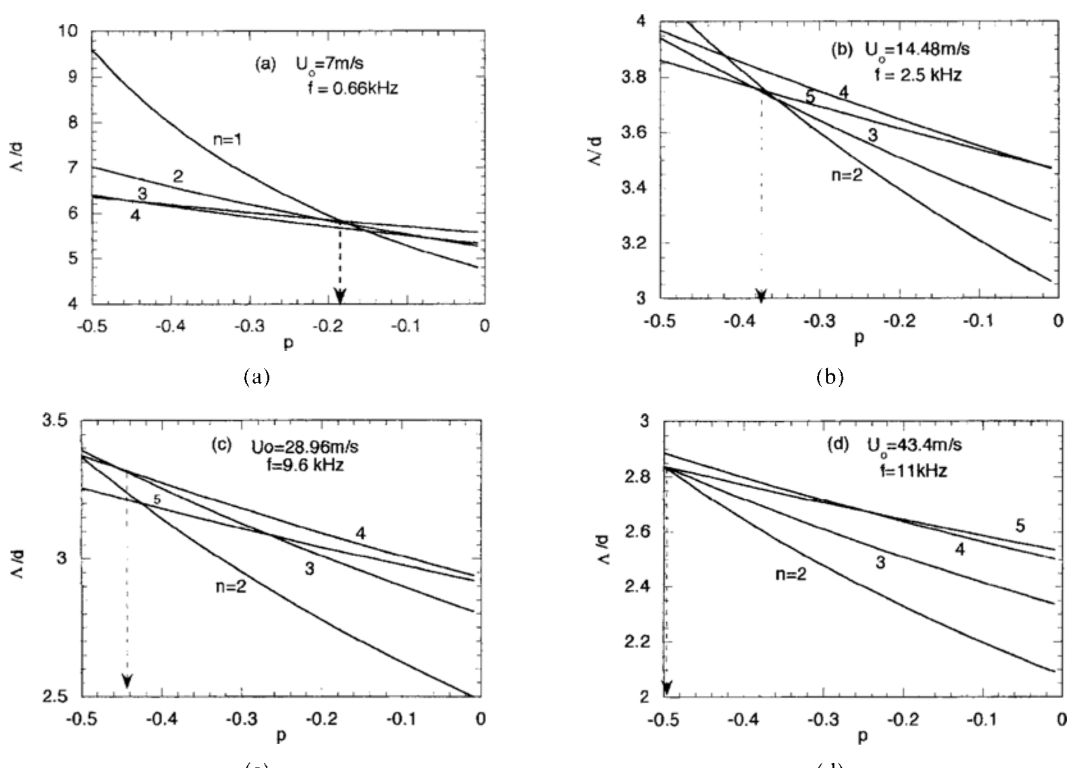


**Fig. 7** Convection wavelength vs. phase factor estimated using the data of Powell and Unfried (1964) for a nozzle slit of width 1.02mm and breadth 12.7mm.

that the curves of different stages do not cross exactly at a point, indicating that the phase factor varies a little depending upon the stage number, that is, the stand-off distance, probably because of the flow breadth or the inaccuracy of the edge alignment with increase in the stand-off distance. So, the deviation is serious when the stage number is the largest. If the case of the highest stage number is eliminated, however, the curves cross almost at a single point and the phase factor can be found as shown by an arrow mark. The phase factor $p$ is shown to decrease from $-0.18$ to $-0.5$ as the jet velocity increases. These values are apparently within the range of the theoretical prediction, $-0.5<p<0$. Figure 8 is the estimated wavelength for the slit of width 0.99mm and breadth 25.4mm, higher aspect ratio than Fig. 7. In this case it can be found that the phase factor is almost constant around $-0.3$ regardless of the jet velocity. The difference between Fig. 7 and Fig. 8 is the nozzle configuration. Depending on the nozzle, the velocity pattern may differ and thereby the effective source point can be influenced. In Fig. 7 the phase factor is shown to decrease with the jet velocity or the convection wavelength. If we assume the effective source point is nearly invariant, the phase factor corresponding to the ratio of the effective source point to the convection wavelength should be changed as in Fig. 7. However, such trend as in Fig. 7 can not be found in Fig. 8. In short, the phase factor can not be a universal constant and varies in the range $-0.5<p<0$ depending upon many parameters associated with the flow behaviour in the process of jet-edge interaction. Hence $p=-1/4$ may be regarded as the approximate mean value.

The phase factor may be interpreted to represent the phase delay associated with the time for the disturbance to travel around between the edge tip and the effective source point. Hence there should be an integer number of waves, $n$, in the feedback loop over the distance $h'$ between the

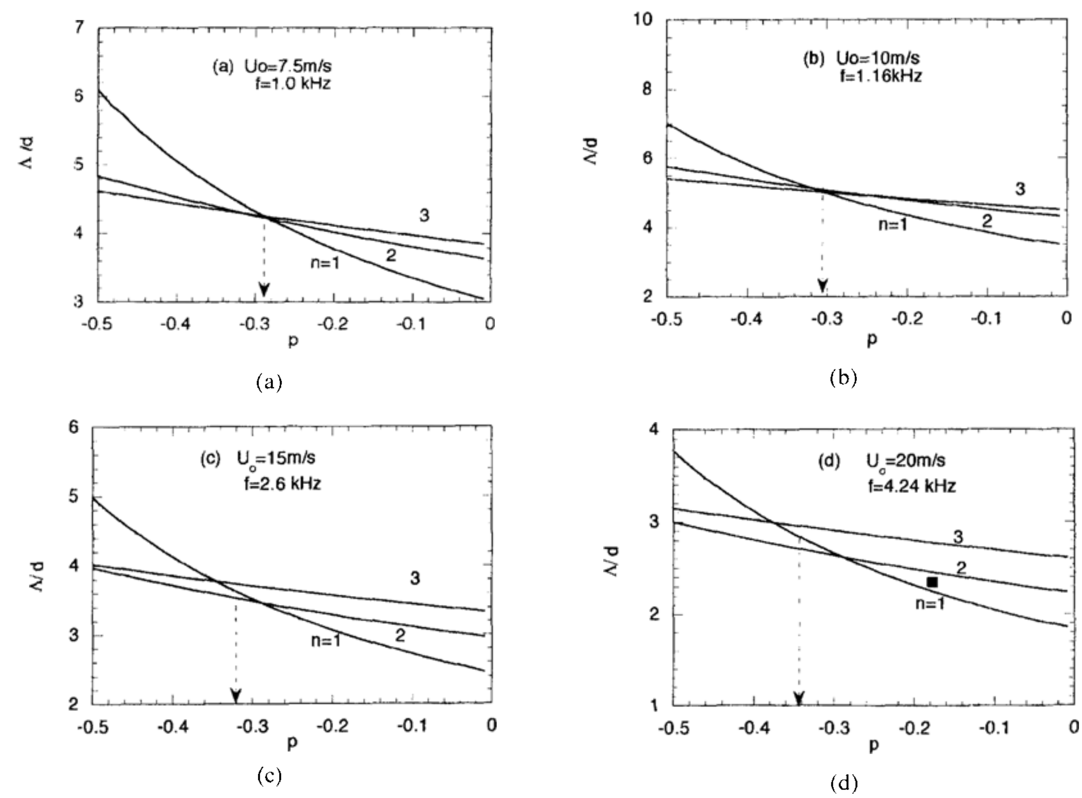


**Fig. 8** Convection wavelength vs. phase factor estimated using the data of Powell and Unfried (1964) for a nozzle slit of width 0.99mm and breadth 25.4mm.

nozzle lip and the effective source point, as discussed previously (Kwon[a] 1996), to be written by

$$\frac{h'}{\Lambda}+\frac{h'}{\lambda}=n \tag{18}$$

If the convection Mach number, $M_c=\frac{\Lambda}{\lambda}=\frac{U_c}{c}$, is low enough, the nonintegral factor $p$, associated with the phase criterion of Eq. (1) can be written as

$$p=-\frac{h'-h}{\Lambda} \tag{19}$$

The above relation suggests that the phase factor is associated with the distance of the effective source point from the edge tip divided by the convection wavelength.

## 5. Conclusion

The feedback mechanism of low-speed edgetones has been studied by using a simple model of jet-edge interaction where the edge reaction is regarded as an array of dipoles. By assuming the downstream-convected disturbances as a sinuously fluctuating flow with a constant convection speed, the strength of the dipoles and then the pressure distribution on the edge surface have been estimated and the maximum pressure point, to be regarded as the effective source point, has been found to be within half a wavelength downstream from the edge tip. Based on the edge reaction, the upstream wave has been estimated in terms of its magnitude and phase. From the phase relationship the phase factor was found to be associated with the effective source point and to be in the range, $-1/2<p<0$, different depending on the flow behaviour in the process of the jet-edge interaction and confirmed in comparison with existing experimental data.

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